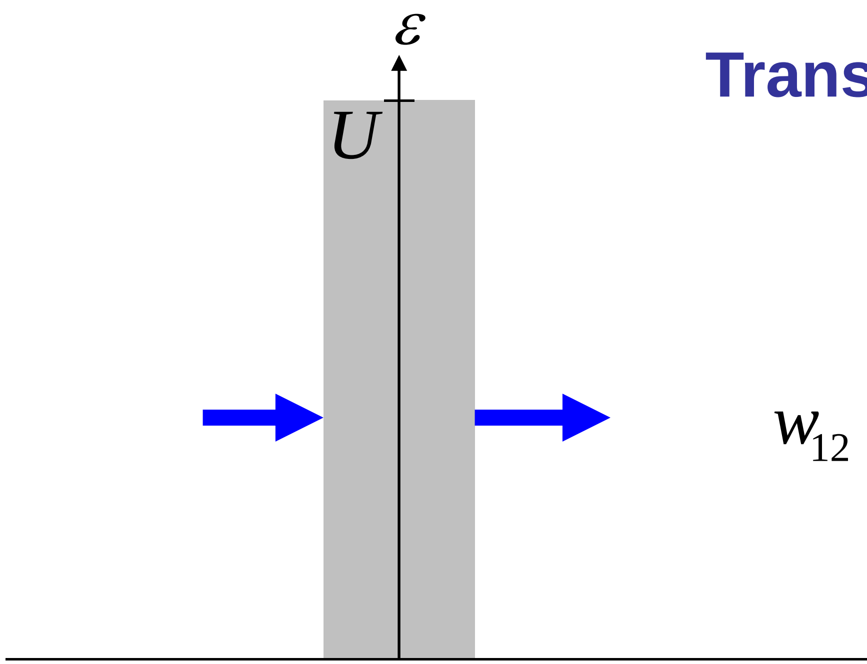
Lectures 22-24

The resonance tunnelling phenomenon

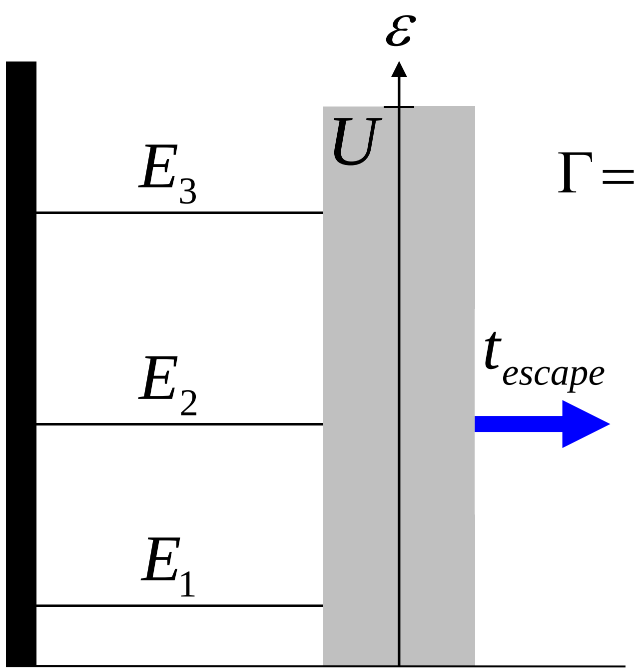
The Coulomb blockade effect
and single-electron transistor (SET)



Transmission through a barrier

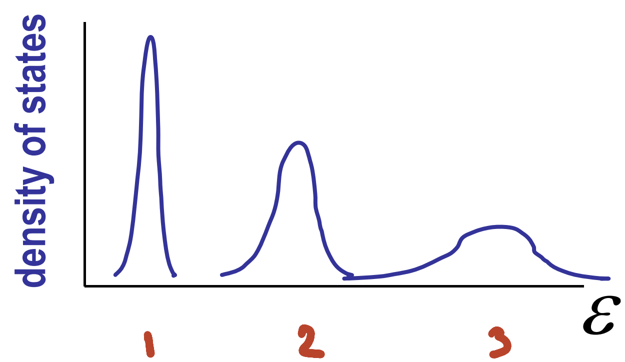
Probability to pass

$$w_{12} = \exp\left\{-2\frac{d}{\hbar}\sqrt{2m_e(U-\varepsilon)}\right\} = e^{-k^*d}$$



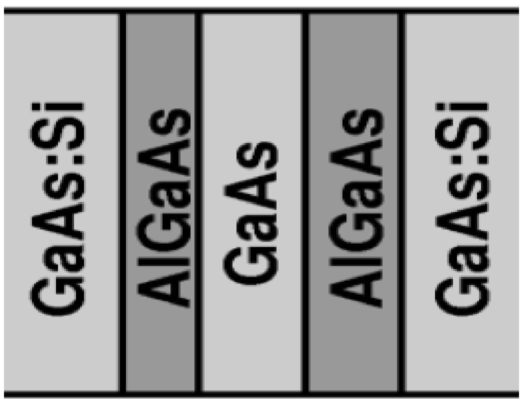
Escape from a confined state and 'level broadenning'.

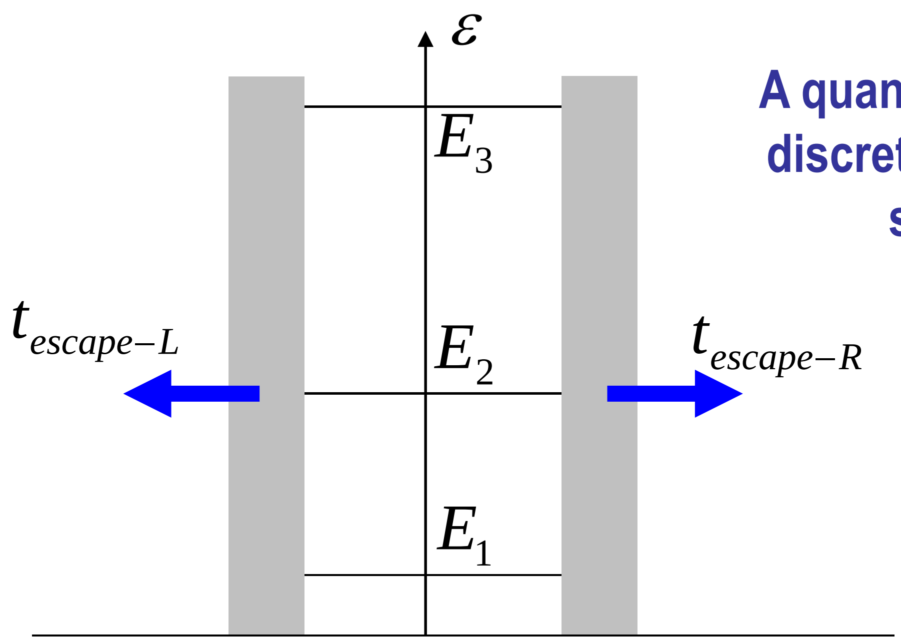
$$\Gamma = \frac{h}{t_{escape}} \sim h\omega_{attempts}\exp\left\{-2\frac{d}{\hbar}\sqrt{2m_e(U-\varepsilon)}\right\} \sim h\omega_{attempts}e^{-k^*d}$$



Level broadening:
due to a finite life-time
of electron on the dot,
its energy is not
known with accuracy
better than Γ

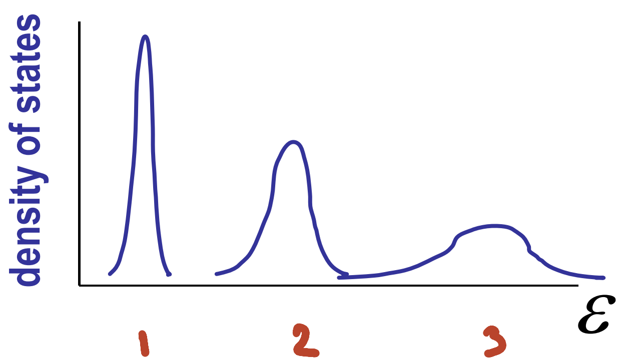
Double-barrier structure





A quantum well has a discrete spectrum of states E_n

$$\Gamma_L = \frac{h}{t_{escape-L}} \sim h\omega e^{-k^* d_L} \qquad \Gamma_R = \frac{h}{t_{escape-R}} \sim h\omega e^{-k^* d_R}$$



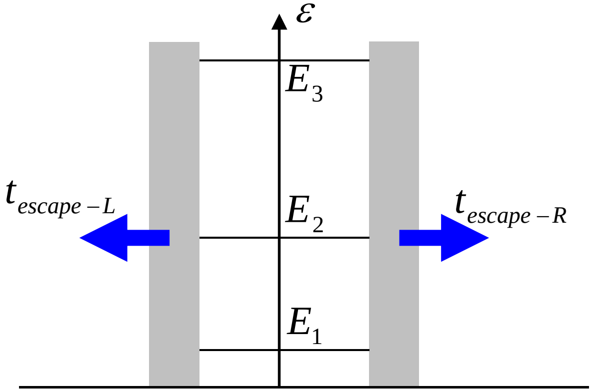
$$\Gamma_L + \Gamma_R = \Gamma$$

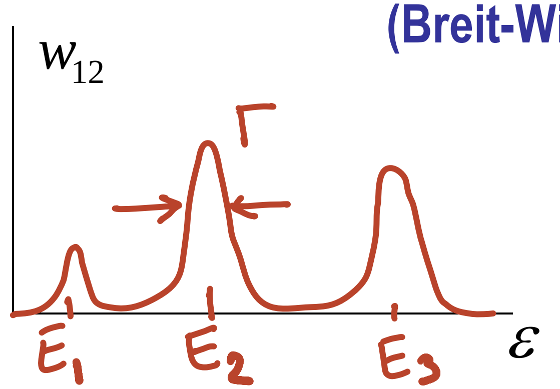
Level broadening is determined by the electron escape to both (left and right) reservoirs.

Resonance tunnelling phenomenon and transmission through double-barrier structures

$$w_{12}(\varepsilon) \approx \frac{\Gamma_R \Gamma_L}{(\varepsilon - E_n)^2 + \frac{1}{4}(\Gamma_R + \Gamma_L)^2}$$

(Breit-Wigner formula)





$$w_{12}(\varepsilon) = \begin{cases} \dfrac{4\Gamma_R \Gamma_L}{(\Gamma_R + \Gamma_L)^2} = 1 & \text{if} \quad \varepsilon = E_n, \quad \Gamma_L = \Gamma_R \\ \dfrac{4\Gamma_R \Gamma_L}{(\Gamma_R + \Gamma_L)^2} \sim \dfrac{4e^{-k^*(d_L + d_R)}}{(e^{-k^* d_L} + e^{-k^* d_R})^2} \sim 4e^{-k^*|d_L - d_R|} & \text{if} \quad \varepsilon = E_n, \quad d_L \neq d_R \\ \dfrac{4\Gamma_R \Gamma_L}{(\varepsilon - E_n)^2} \sim e^{-k^*(d_L + d_R)} \to 0 & \text{if} \ |\varepsilon - E_n| >> \Gamma \end{cases}$$

Resonant tunneling in double-barrier semiconductor structures

discovered by Chang, Esaki, Tsu in 1974
(IBM - Watson Research Center)

Leo Esaki
The Nobel Prize in Physics 1973

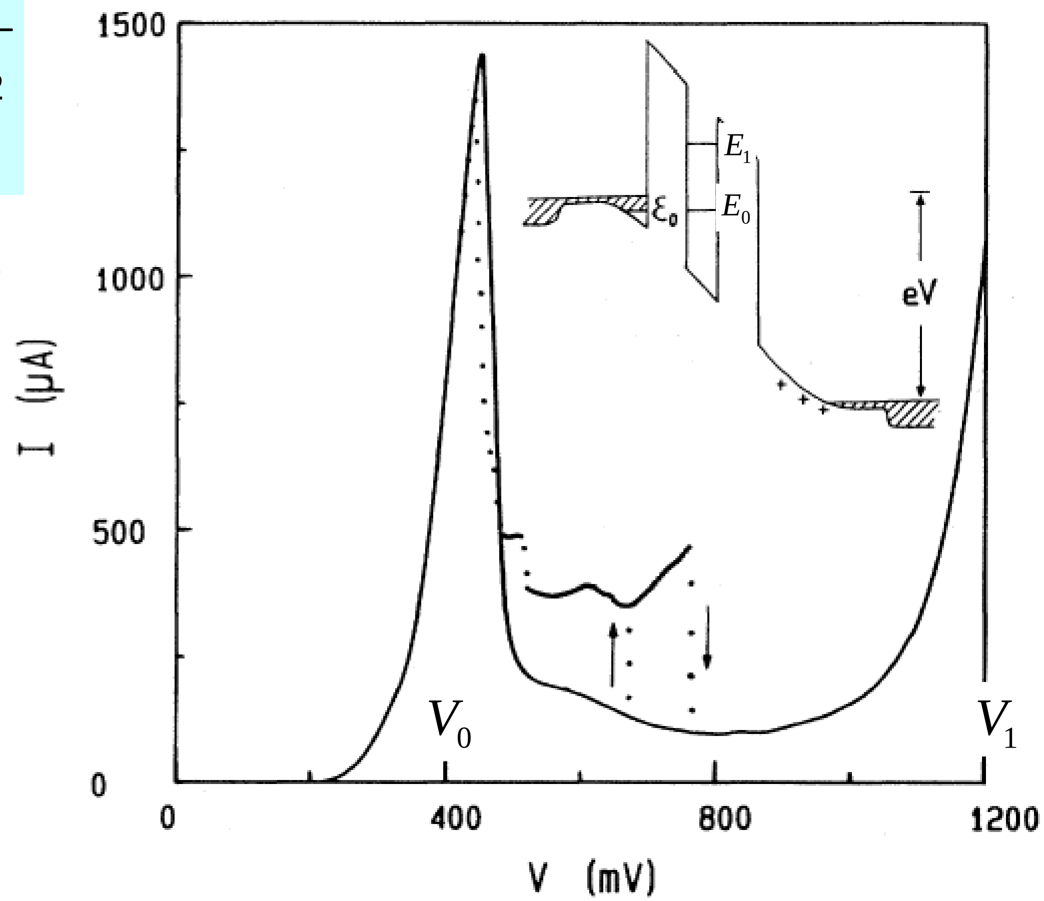
$$I \sim w_{12} = \frac{\Gamma_R \Gamma_L}{(\varepsilon_0 - E_n)^2 + \frac{1}{4}(\Gamma_R + \Gamma_L)^2}$$

For tunnelling from a two-dimensional emitter (quantum well or heterostructure) through a planar double-barrier structure, in-plane momentum conserves throughout tunnelling process. Then the resonant condition,

$$\varepsilon_0 + \frac{p_\parallel^2}{2m} = E_n + \frac{p_\parallel^2}{2m}$$

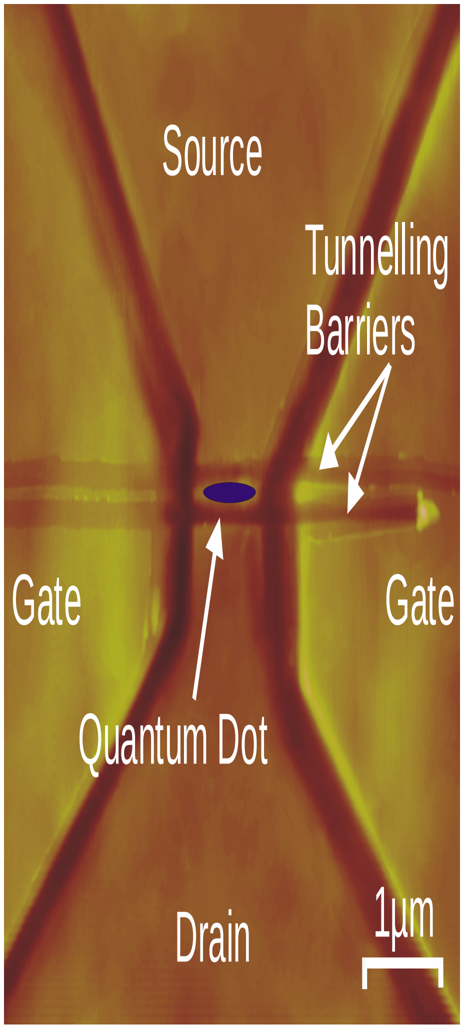
is the same for all electrons in the 2D emitter, and the current has a peak when

$$\varepsilon_0 = E_n(V_n)$$

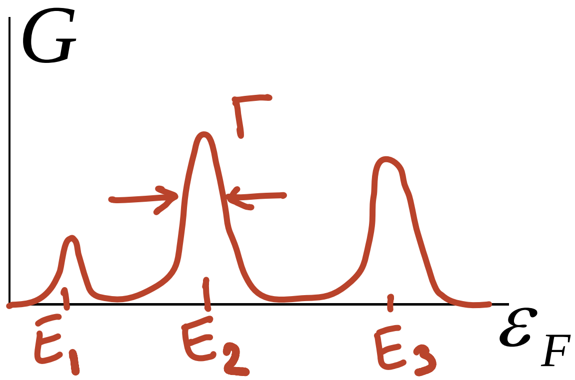


This plot has been taken from publications of L Eaves and Nottingham semiconductor group.

Resonance transmission through a quantum dot



$$G = \frac{2e^2}{h} w_{12}(\varepsilon_F)$$

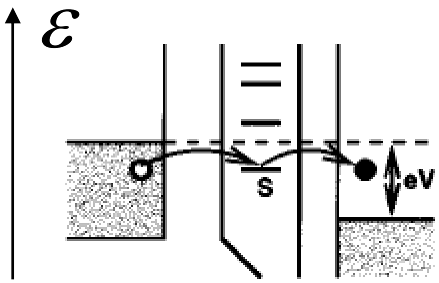


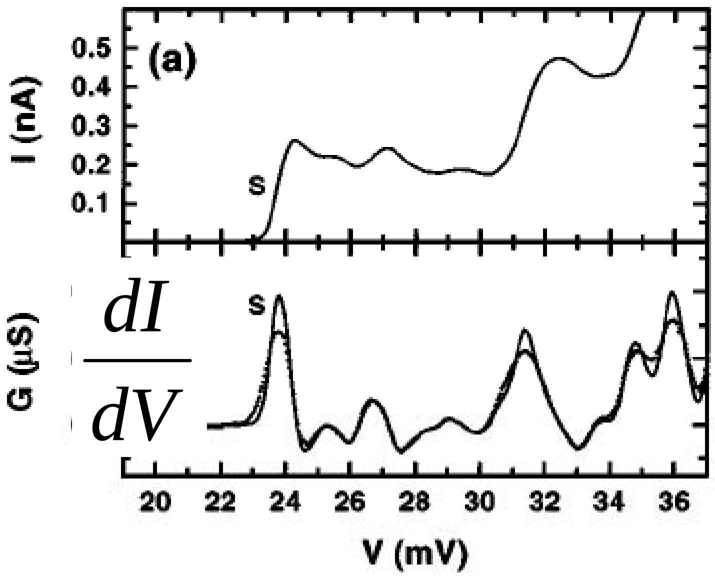
$$w_{12}(\varepsilon_F) = \frac{\Gamma_R \Gamma_L}{(\varepsilon_F - E_n)^2 + \frac{1}{4}(\Gamma_R + \Gamma_L)^2}$$

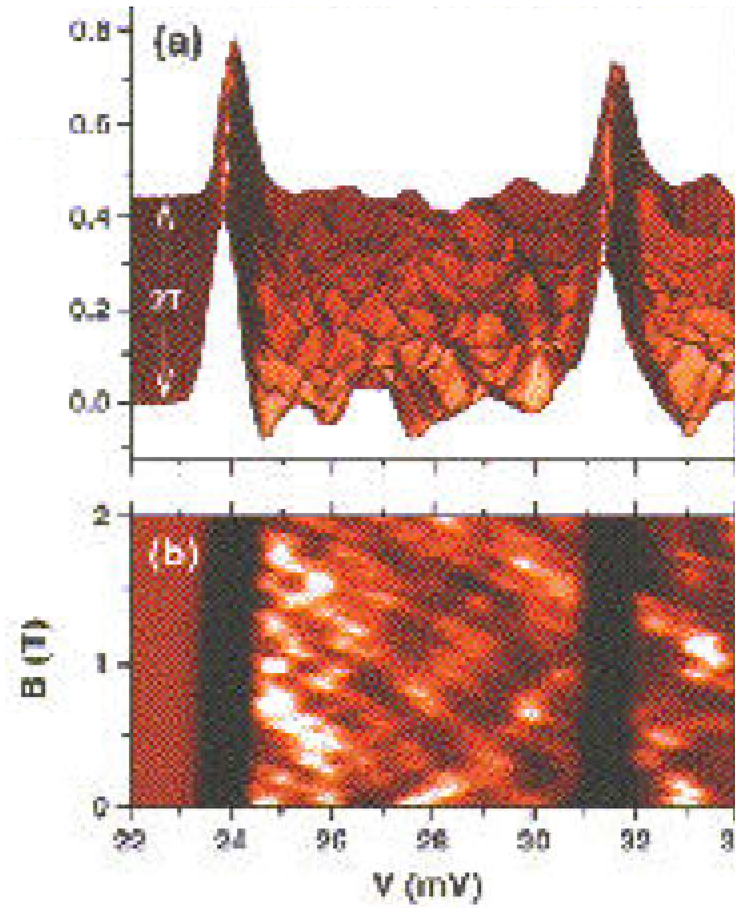
$$G = \begin{cases} \frac{e^2}{h} \frac{4\Gamma_{R-dot}\Gamma_{L-dot}}{(\Gamma_{R-dot} + \Gamma_{L-dot})^2} \sim \frac{e^2}{h} & if \quad \varepsilon_F = E_n \\ 0 \quad if \ |\varepsilon_F - E_n| > \Gamma \end{cases}$$

Resonant tunnelling through a quantum dot

$$G_{diff}(V) = \frac{dI}{dV}(units\ \tfrac{e^2}{h})$$







Groups of K. von Klitzing (MPI-Stuttgart) and R. Haug (Hannover)

Buttiker-Landauer formula – generalisation to finite temperatures.

$$I = 2e\int_0^{\infty}\frac{dp}{h}[f_L(\varepsilon_p) - f_R(\varepsilon_p)]v(p)w_{12} = V\frac{2e^2}{h}\int_0^{\infty}d\varepsilon\left(-\frac{df}{d\varepsilon}\right)w_{12}(\varepsilon)$$

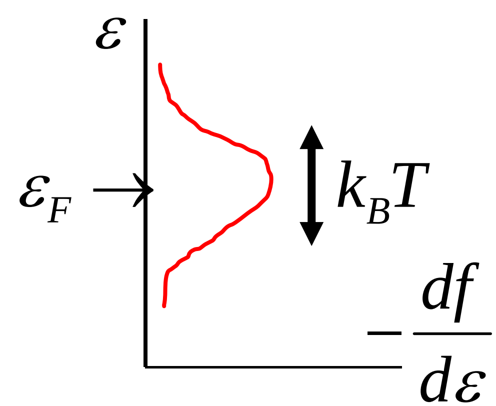
$$f_{L/R}(\varepsilon) = f(\varepsilon, \varepsilon_F \pm \tfrac{1}{2}eV)$$

Fermi distribution function

$$f(\varepsilon, \varepsilon_F) = \frac{1}{1 + e^{(\varepsilon - \varepsilon_F)/k_BT}}$$

$$f_L(\varepsilon) - f_R(\varepsilon) \approx -eV\frac{df}{d\varepsilon}$$

$$\frac{df}{d\varepsilon} = -\frac{e^{(\varepsilon-\varepsilon_F)/k_BT}}{[1 + e^{(\varepsilon-\varepsilon_F)/k_BT}]^2} = \frac{-\frac{1}{2}}{1 + \cosh\frac{2(\varepsilon - \varepsilon_F)}{k_BT}}$$

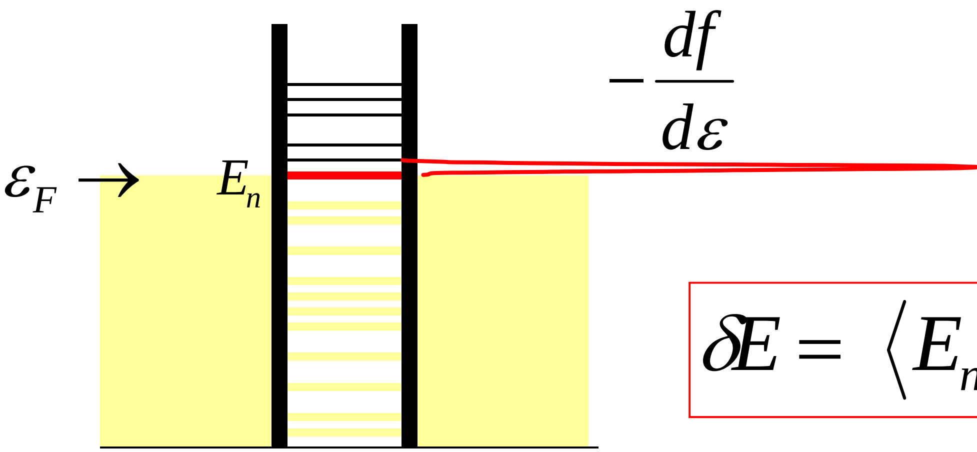


Resonance tunnelling through a 'multi-level' quantum dot

$$G = \frac{2e^2}{h}\int_0^\infty d\varepsilon\left(-\frac{df}{d\varepsilon}\right)w_{12}(\varepsilon)$$

$$w_{12}(\varepsilon) = \sum_n \frac{\Gamma_R\Gamma_L}{(\varepsilon - E_n)^2 + \frac{1}{4}(\Gamma_R + \Gamma_L)^2}$$

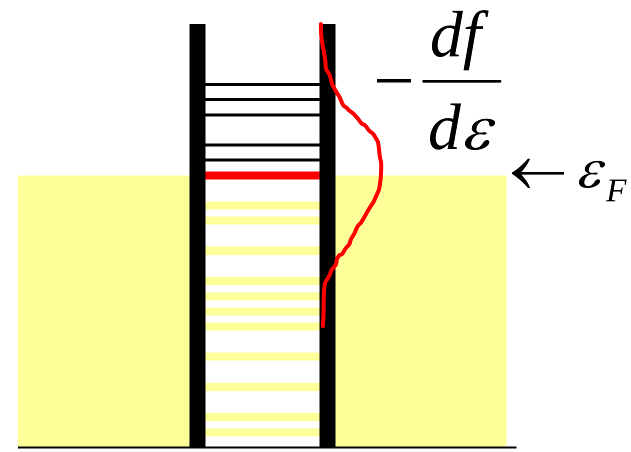
$T \to 0$



$$G = \frac{2e^2}{h}\frac{\Gamma_R\Gamma_L}{(\varepsilon_F - E_n)^2 + \frac{1}{4}(\Gamma_R + \Gamma_L)^2}$$

$$\delta E = \langle E_n - E_{n-1} \rangle$$

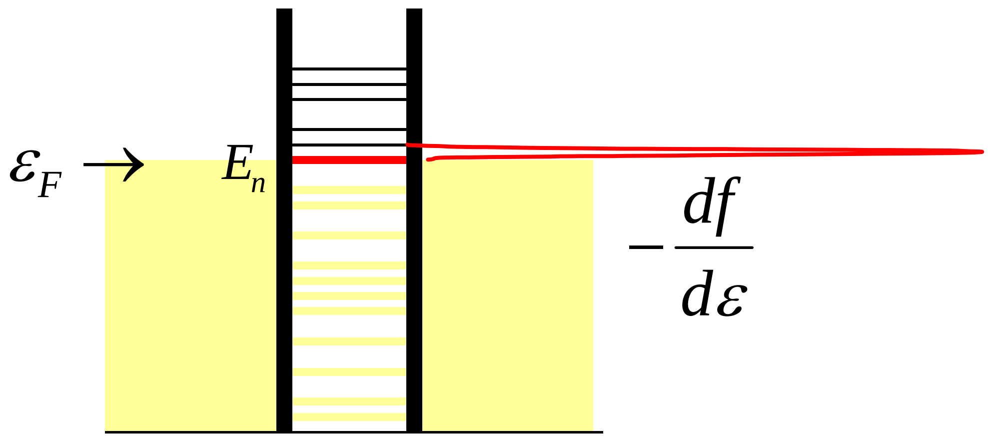
$k_B T >> \delta E$



$$G \sim \frac{e^2}{h}\frac{\Gamma_R\Gamma_L}{(\Gamma_R + \Gamma_L)\delta E}$$

$$\sim \frac{e^2}{h}\frac{\Gamma}{\delta E} \propto e^{-k^* d}$$

Conditions necessary for the observation of resonance tunnelling through a 'multi-level' quantum dot.



$$-\frac{df}{d\varepsilon}$$

$$k_B T < \delta E$$

$$\delta E = \langle E_n - E_{n-1} \rangle$$

$$\delta E \sim \frac{\varepsilon_F}{N} \rightarrow \begin{cases} \dfrac{\varepsilon_F}{L/\lambda_F} \sim \dfrac{\hbar v_F}{L} & L \\ \dfrac{\varepsilon_F}{(L/\lambda_F)^2} \sim \dfrac{\hbar^2/m_e}{L^2} & L \times L \\ \dfrac{\varepsilon_F}{(L/\lambda_F)^3} \sim \dfrac{\lambda_F \hbar^2/m_e}{L^3} & L \times L \times L \end{cases}$$

quickly decreases with size (for $L \times L$ and $L \times L \times L$)

The Coulomb blockade

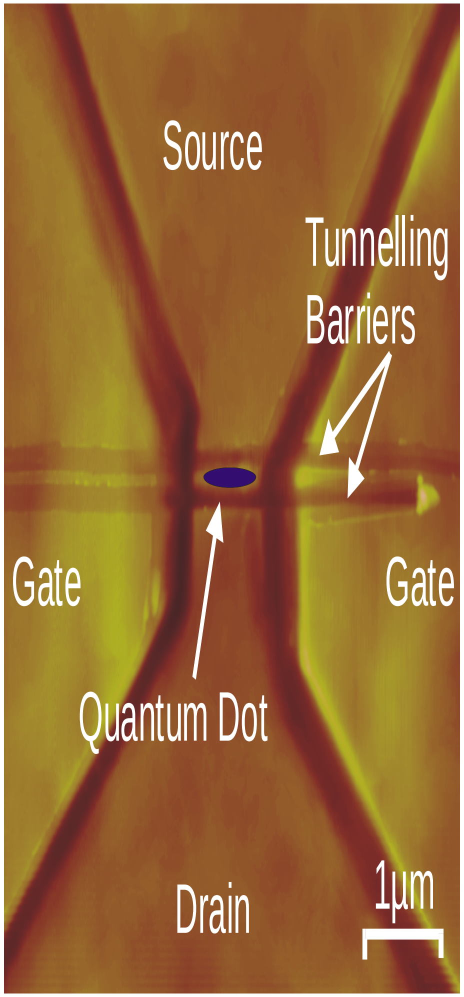
Dynamical screening and Coulomb blockade in quantum dots.

Counting electrons one by one.

Single-electron transistor (SET)

Coulomb blockade in a superconducting island: ‘parity effect’.

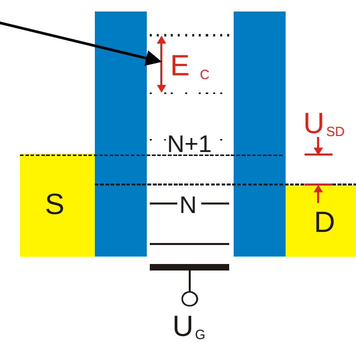
Charge quantization in isolated quantum dots

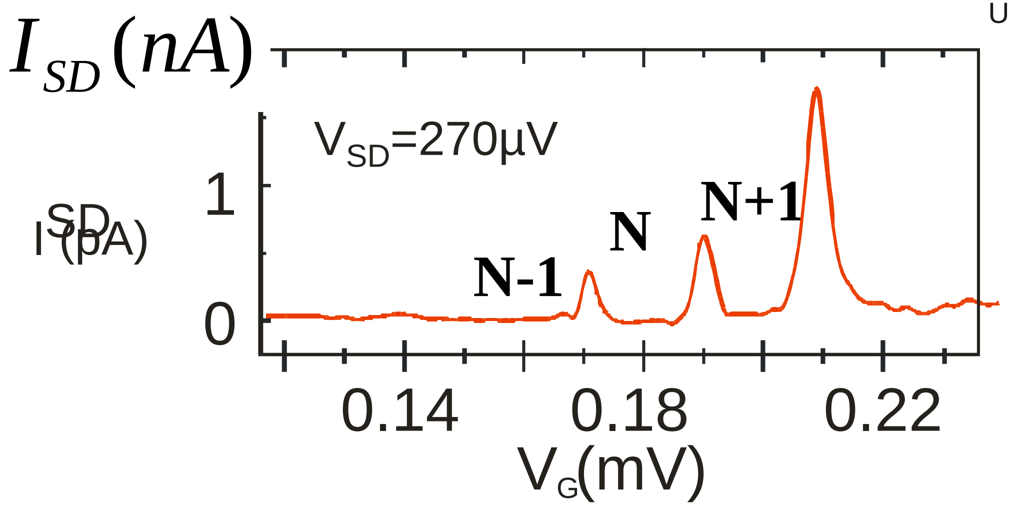


$$E_c = \frac{e^2}{2C}$$

$$C \sim 4\pi\varepsilon_0 L$$

Coulomb blockade



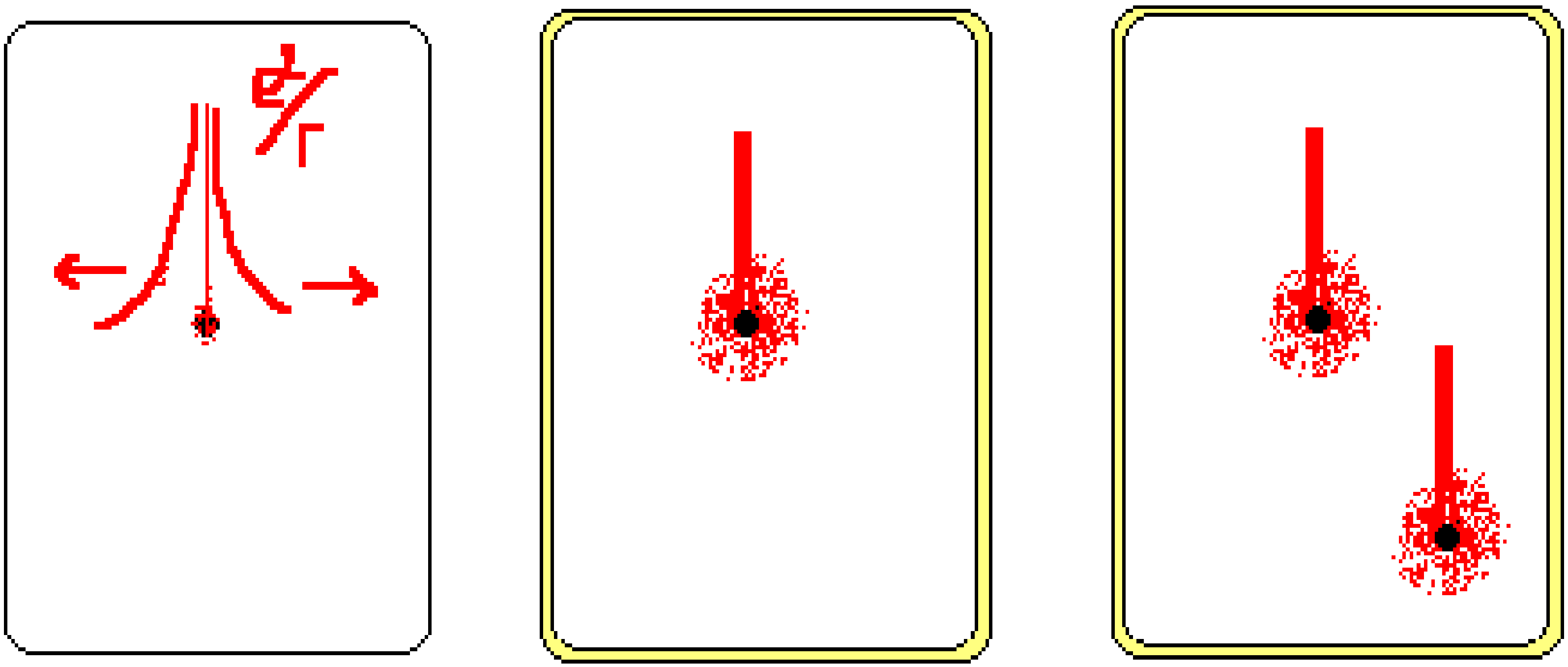


<u>First observations</u>: **T.Fulton, G.Dolan PRL 59, 109 (1987) – Bell Labs**
M.Kastner - Rev. Mod Phys. 64, 849 (1992) – MIT

<u>Active groups</u>:
Marcus (Harvard); Kouwenhoven TUDelft); Haug (Hannover); Enssling (ETH Zurich)

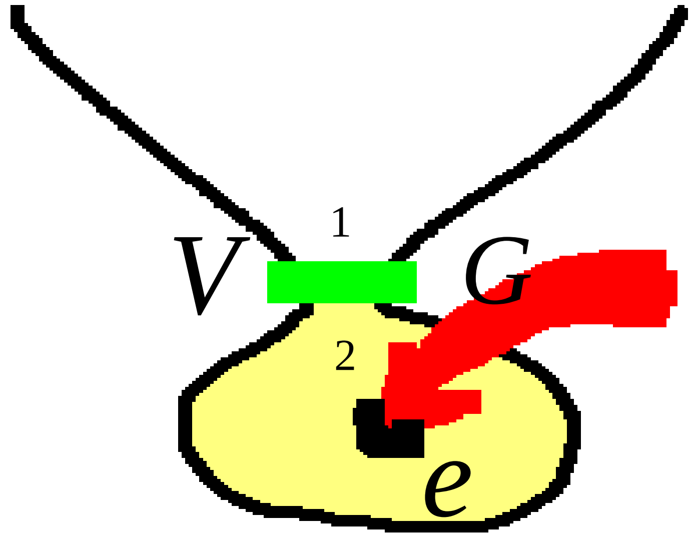
Charge quantization

$$V_{ee}(\vec{r}_1 - \vec{r}_2) = F \cdot \delta(\vec{r}_1 - \vec{r}_2)$$



Although electron carries electric charge e, its interaction with other electrons is screened by the other electrons from the Fermi sea, so that e-e interaction is reduced.

$$\frac{dQ}{dt} = -I = -GV = -\frac{GQ}{C}$$



$$\tau_{scr}^{-1} = \frac{G}{C} = \frac{2e^2 w_{12}}{hC}$$

Decay rate of charge localised at the dot determines broadening of single-electron charged state of the dot due to dynamical screening

Charging energy

$$E_c = \frac{e^2}{2C} > h\tau_{scr}^{-1} = \frac{2e^2 w_{12}}{C}$$

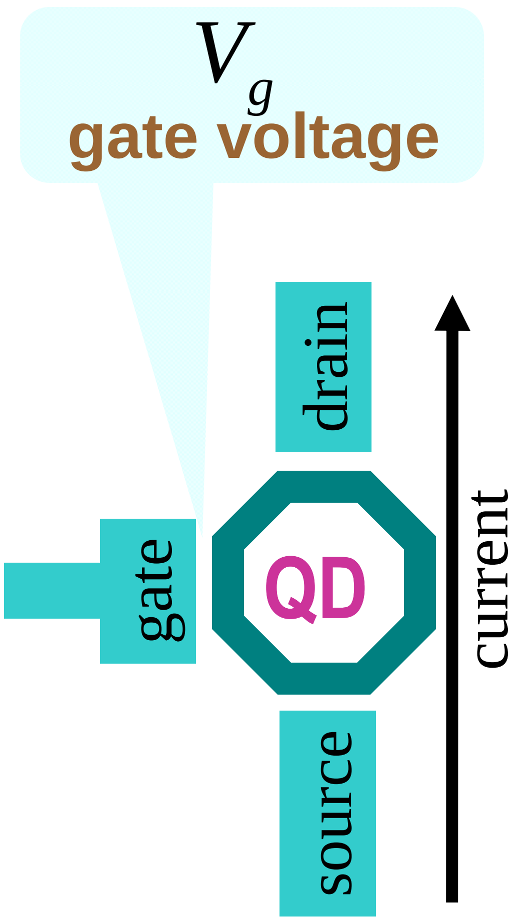
if

$$w_{12} < 1$$

$$Q = Ne$$

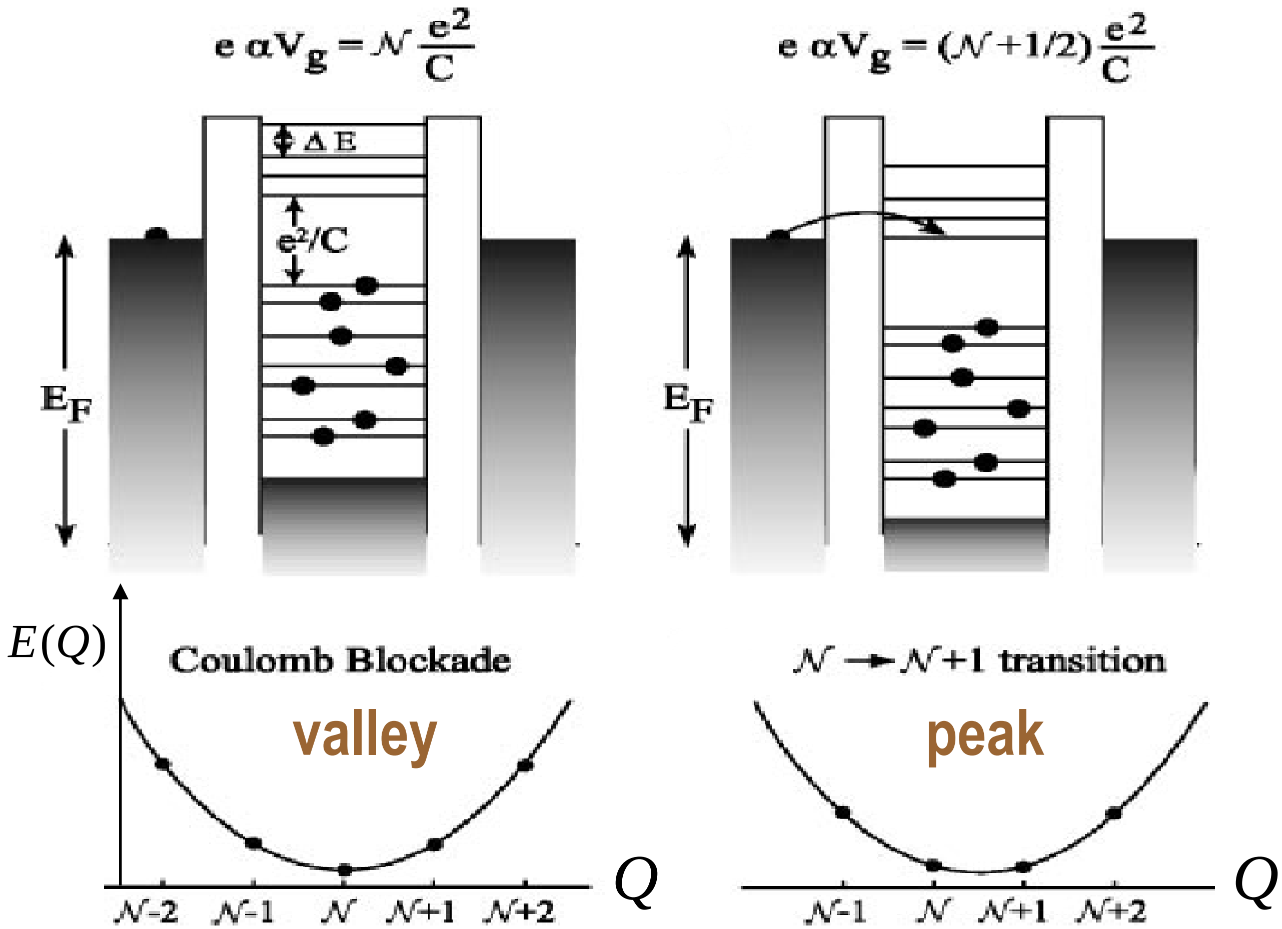
screening is blocked: Charge is quantized

Coulomb Blockade of electron tunneling

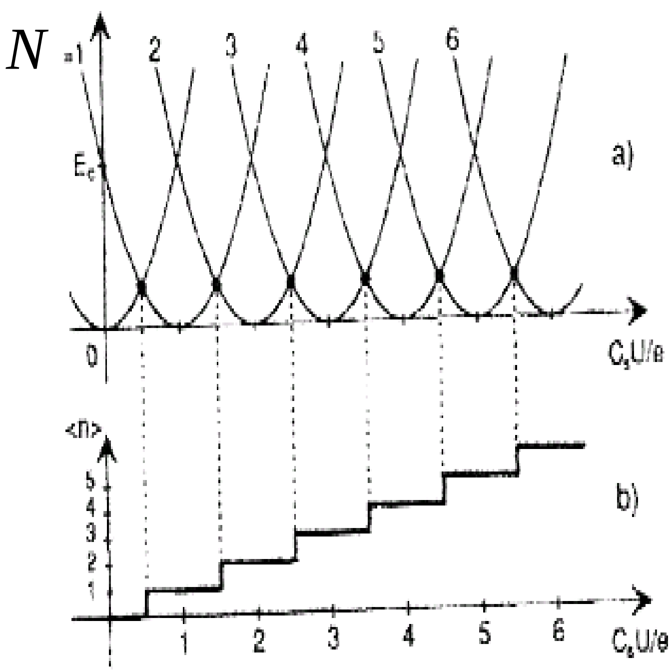


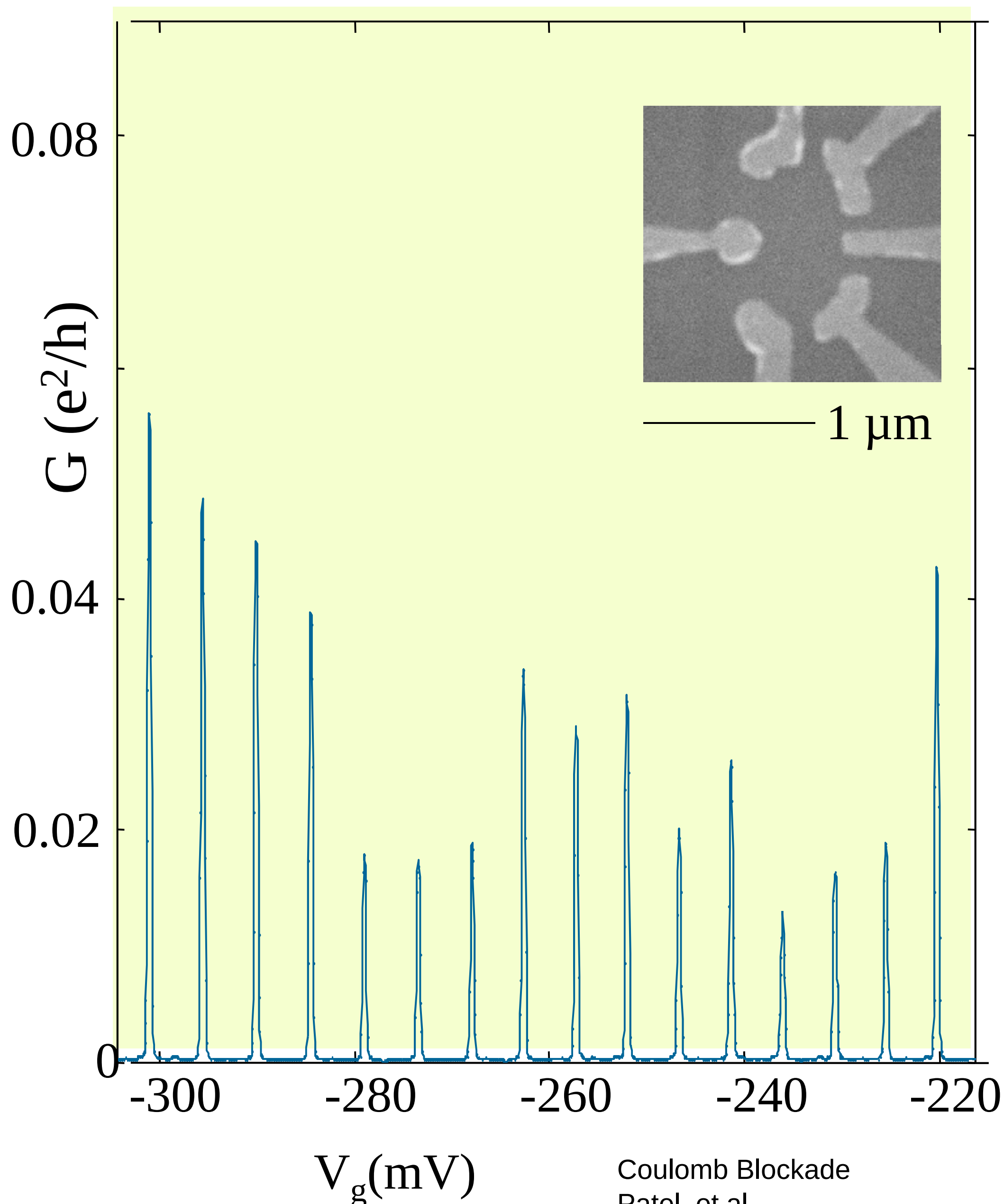
$$E = \frac{(Q - CV_g)^2}{2C}$$

$$Q = Ne$$



$$E_N(V_g) = \frac{(Ne - CV_g)^2}{2C}$$



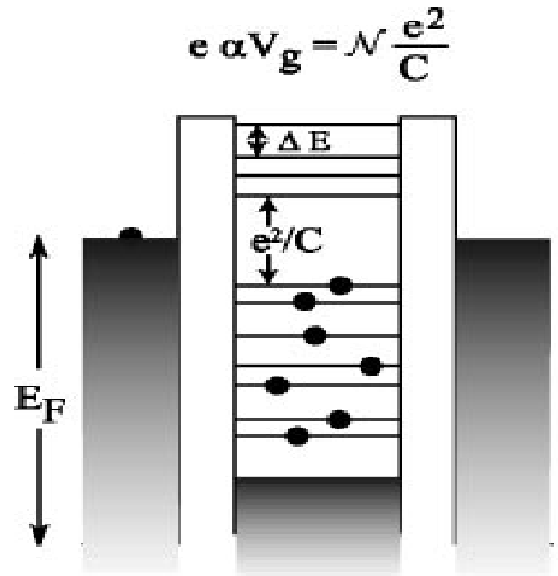


Coulomb Blockade
Patel, et al.
Phys.Rev.Lett. 80 4522 (1998)

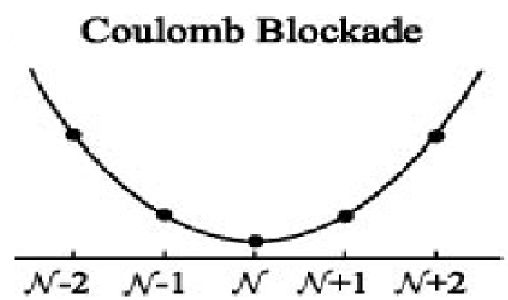
Conditions for the observation/use of the Coulomb Blockade of electron tunneling:

$$E_c = \frac{e^2}{2C} >> k_B T$$

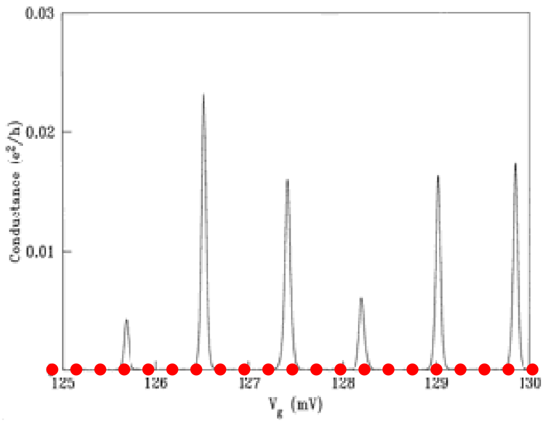
Low noise due to the environment.

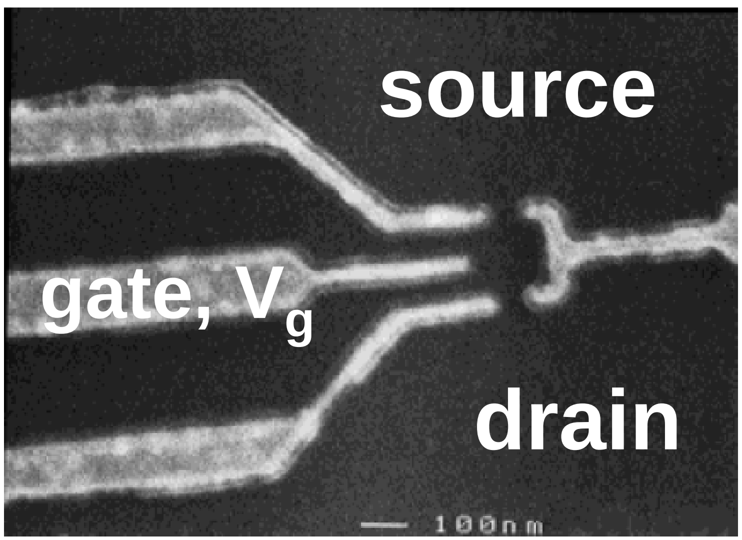


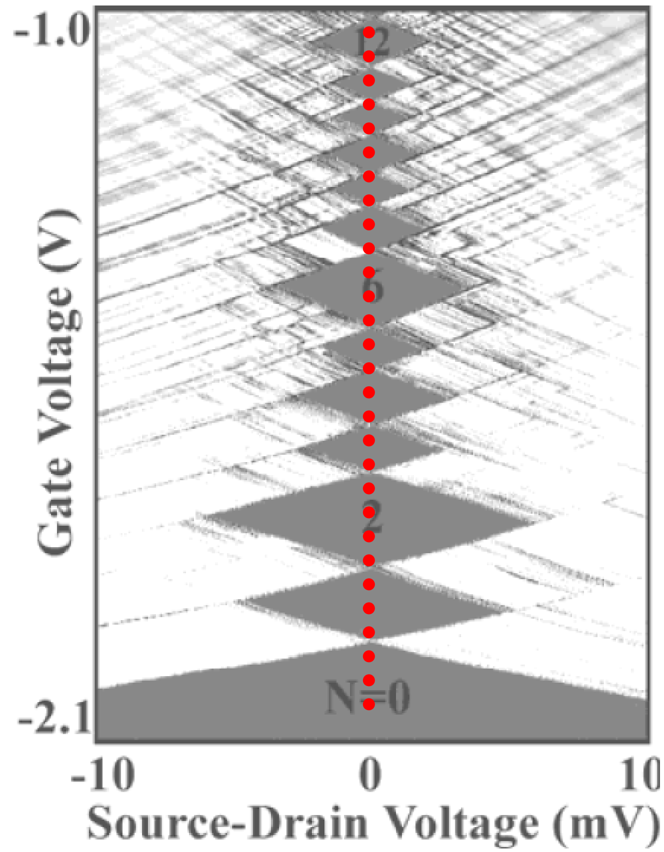
$$E_N(V_g) = \frac{(Ne - CV_g)^2}{2C}$$



Single electron transistor (SET)



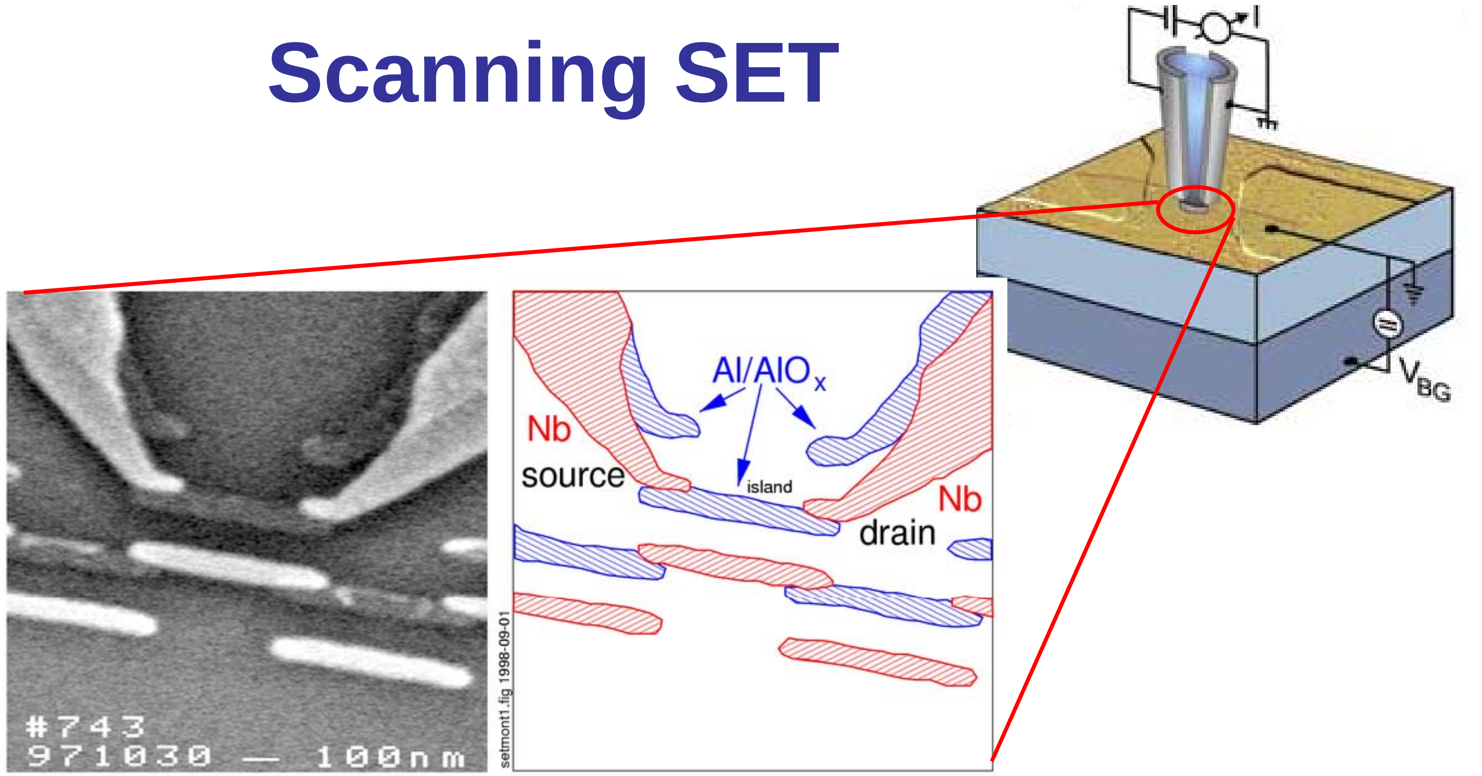




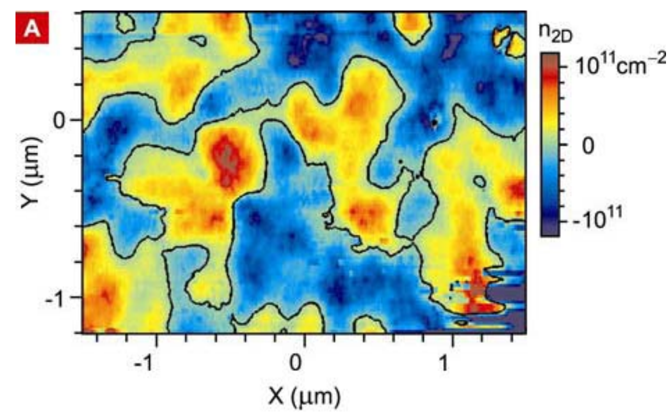
D. Goldhaber-Gordon and M. Kastner (MIT)

Current through a biased SET is very sensitive to the electrostatic environment, so that it can be used to measure distribution of potential/charges on the surfaces.

Scanning SET



Scan of a density distribution in graphene deposited on SiO_2. The observed inhomogeneous electron density (p- and n-type puddles) is due to deposits stuck between the substrate and graphene sheet.

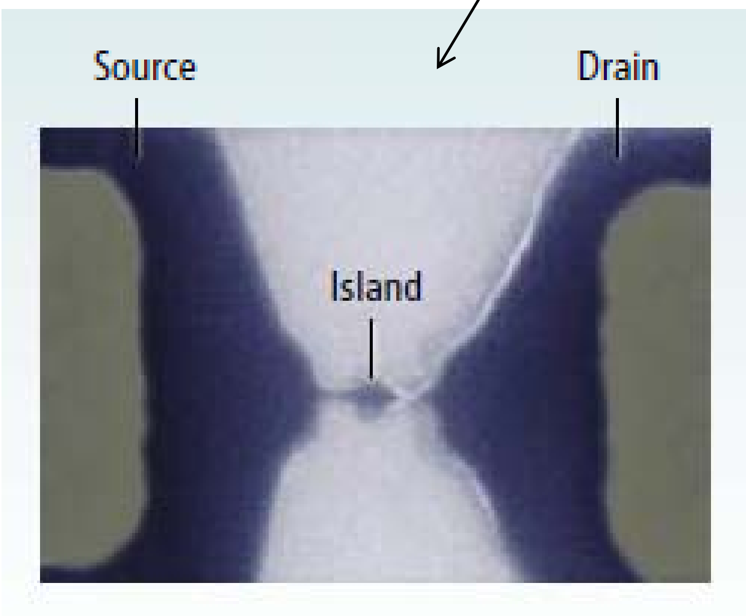


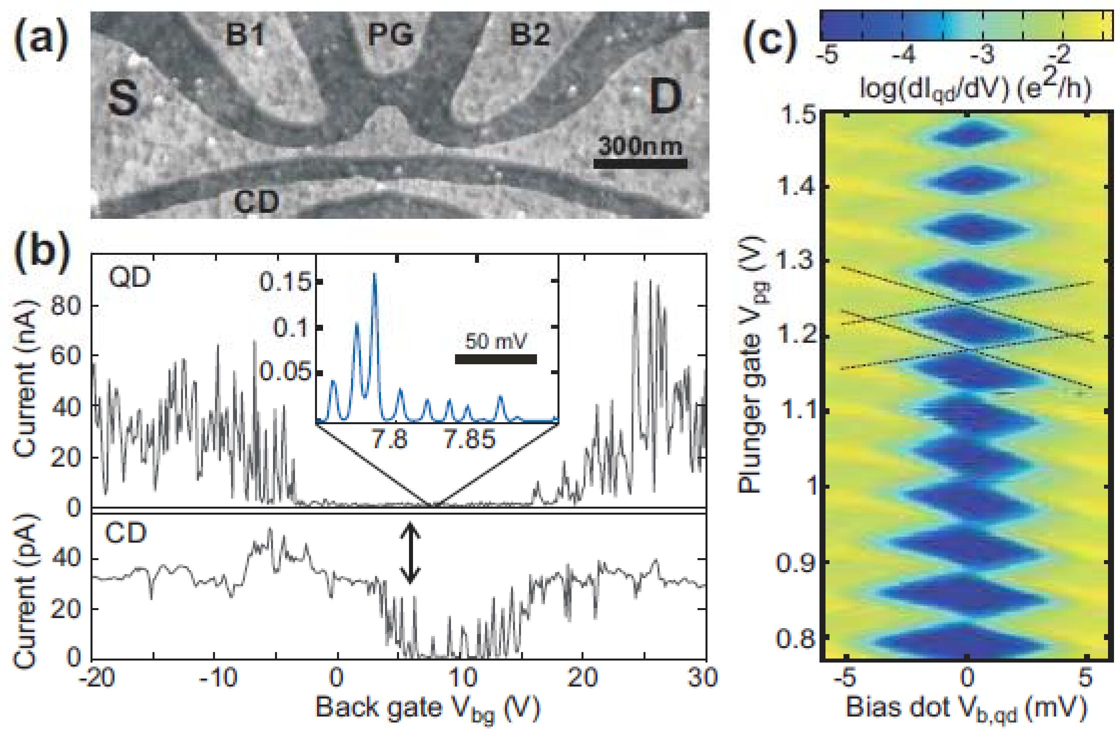
A.Yacobi (Harvard)
Science, 2007

Graphene quantum dot circuits and graphene-based single-electron transistor

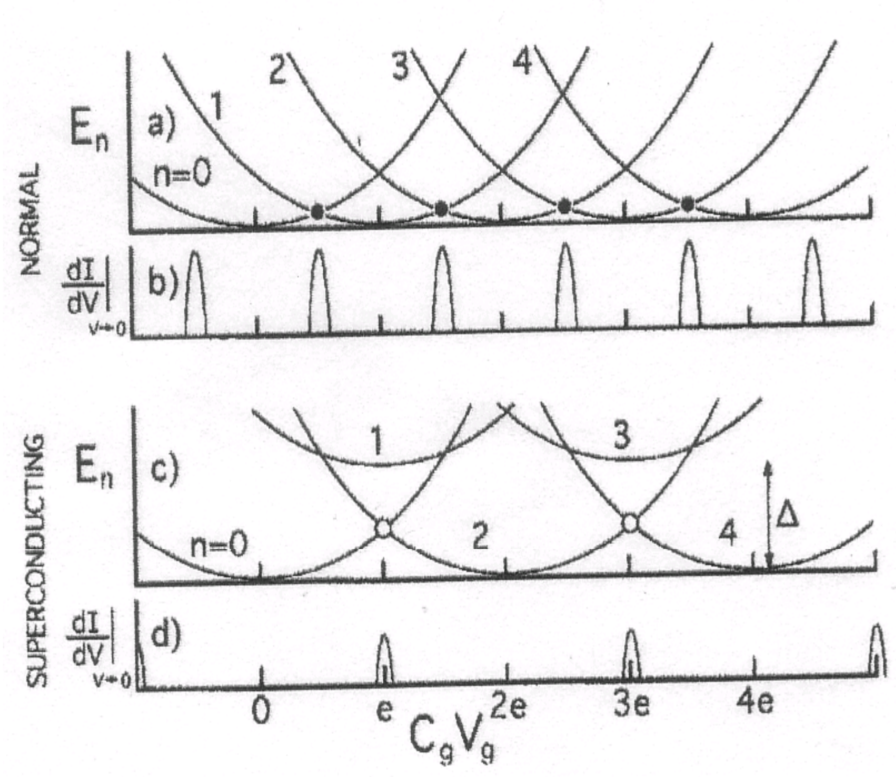
Geim & Novoselov (Manchester)

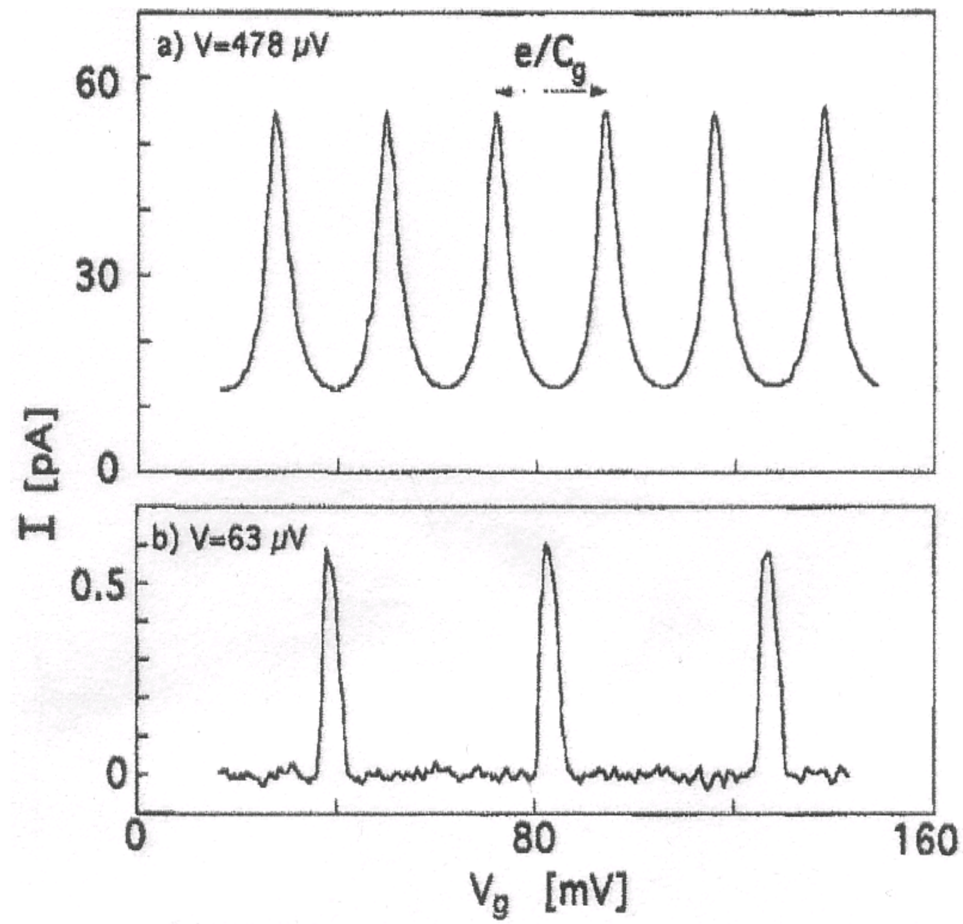
Ensslin (ETH Zurich)



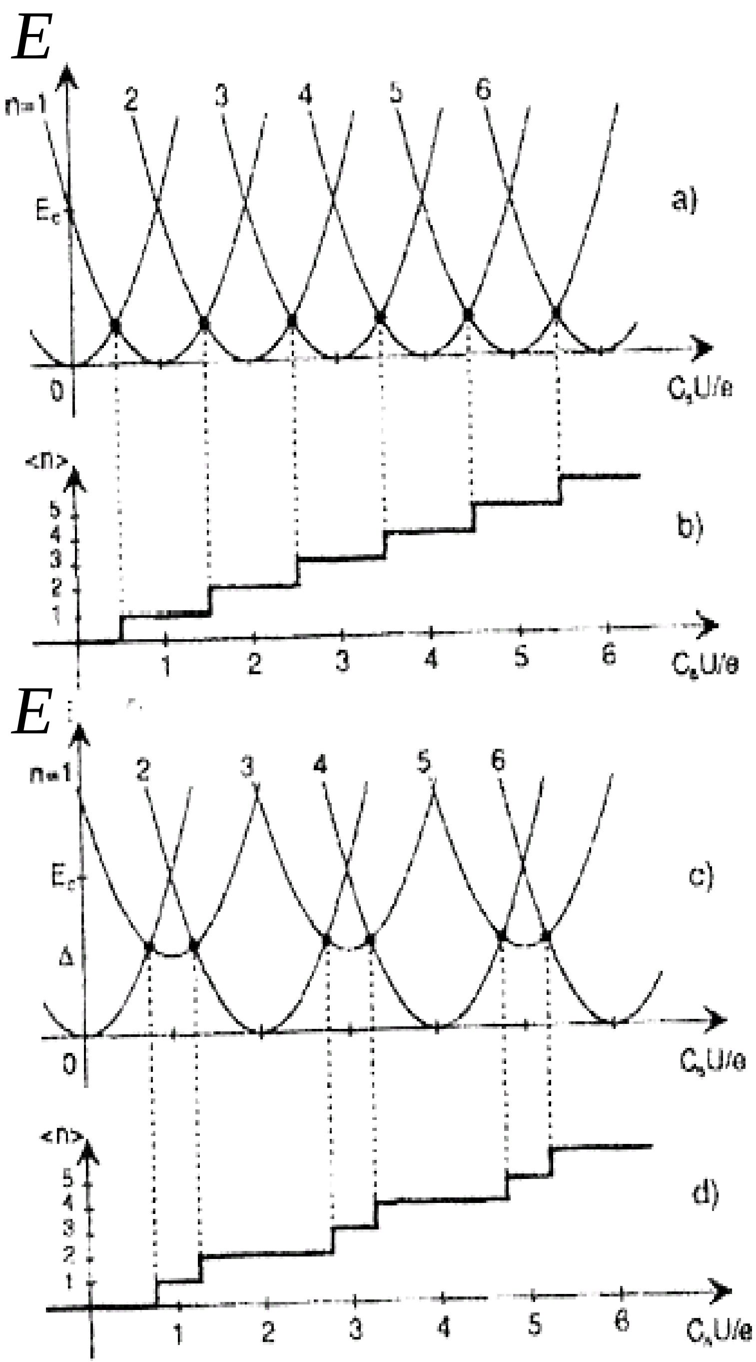


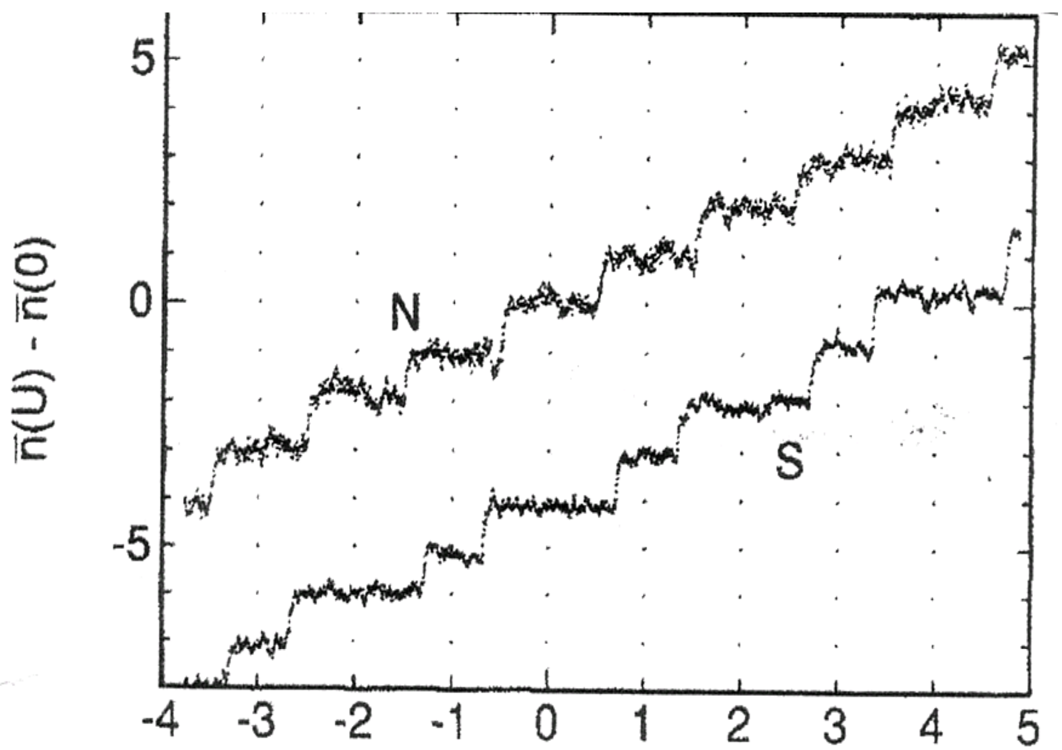
Coulomb blockade in a superconducting island: condensate of Cooper pairs creates a gap Δ in the single-particle spectrum





Parity effect in a superconducting island





Lafarge, Joyez, Esteve, Urbina, Devoret, PRL 70, 994 (1993)